A Minimal (2+1)-Dimensional Entropic Gravity Demonstration

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April 5, 2025

Abstract

We present a simplified entropic-gravity style argument in (2+1)-dimensional spacetime, requiring only two key assumptions: (i) that the boundary at radius r has entanglement entropy proportional to its perimeter, $S(r) \sim r$, and (ii) that the associated temperature scales inversely with r. By treating the incremental energy $\mathrm{d}E$ as $\mathrm{d}E = T\,\mathrm{d}S$ (a thermodynamic first-law relation for the boundary) [1, 2], we derive an energy function $E(r)$ whose radial derivative yields a $1/r$ force law. This concise derivation provides a "door-opening" example of how quantum informational constraints alone can yield a Newton-like force in lower dimensions, supporting the broader hypothesis of emergent gravity from entanglement [3, 4].

1 Introduction

The notion that gravity might emerge from quantum informational principles has received renewed attention, particularly via entropic arguments in which boundary thermodynamics plays a fundamental role [1, 2, 3]. While many demonstrations focus on $(3+1)$-dimensional spacetimes, the key ideas often simplify in lower dimensions. Here, we present a short, self-contained derivation of a "Newton-like" $1/r$ force law in $(2+1)$ dimensions, assuming only that an entangling boundary at radius r has: (i) perimeter-proportional entropy, and (ii) a scale-dependent temperature inversely proportional to r. We show that a thermodynamic first-law ansatz $\mathrm{d}E = T\,\mathrm{d}S$ produces a simple potential $E(r) \propto \ln(r)$, whose radial derivative is a $1/r$ force.

This minimal example illustrates the conceptual pathway from quantum informational bounds to a gravitational-type force, underscoring how little additional structure is needed to see an "entropic gravity" phenomenon in lower dimensions. In particular, our work extends the spirit of earlier results in even lower-dimensional models such as the $(1+1)$-dimensional entropic force demonstration by Mann and Mureika [4], but now adapted to the $(2+1)$-dimensional context.

Physical Picture and Scope. We stress that our $(2+1)$D setup is a conceptual model (analogous to a flat-space holographic screen [1]) rather than a literal physical gravitational system. In (2+1)D Einstein gravity without negative cosmological constant, one does not obtain local gravitational fields from isolated masses; thus, the derived $1/r$ force here should be viewed as emergent from quantum informational degrees of freedom, not classical curvature. This clarifies that we are not conflating the toy construction with real-world (2+1)D gravity; instead, we aim to demonstrate, in a stripped-down setting, how boundary thermodynamics can reproduce a radial force in lower dimensions. Our arguments are motivated by well-known results in black hole thermodynamics and holographic approaches, but we focus on a simpler, flat-space analogy that reveals how minimal assumptions about perimeter-scaling entropy and $1/r$ temperature lead to a gravitational-type force.

2 Setup: A (2+1)-Dimensional Boundary

We consider a $(2+1)$-dimensional spacetime with a radial coordinate r running outward from some reference r_0. At each radius r, we posit a circular boundary whose circumference is $\propto r$. In a quantum gravity or holographic context, such a boundary can be thought of as carrying entanglement entropy $S(r)$ [5, 6].

2.1 Perimeter-Law Entropy

Unlike in (3+1)-dimensional black hole horizons, where the area $\sim r^2$ sets the entropy scaling, here the boundary is effectively a circle of length $\propto r$. Hence, we assume the boundary's entanglement entropy is

$$S(r) \;=\; \alpha\, r, \tag{1}$$

where α is a constant with dimensions of (entropy)/(unit length), setting the scale for the entanglement entropy (we use units where $k_{\mathrm{B}} = 1$). This perimeter-law form follows naturally from known (2+1)D black hole horizon results [5, 6, 7] and is consistent with typical boundary ("area-law") entanglement arguments, specialized to one spatial dimension at the boundary.

2.2 Temperature Scaling $\propto 1/r$

Next, we assume the boundary has an associated temperature $T(r)$ that scales inversely with r:

$$T(r) \;=\; \frac{\beta}{r}, \tag{2}$$

where β is a (dimensionless) constant setting the overall temperature scale. While $T(r) = \beta/r$ might seem *a priori* like a convenient guess, it echoes how surface gravity and Hawking-like temperatures can behave in certain lower-dimensional contexts: for instance, in the Unruh effect for an accelerating observer or in analogy with black hole horizons, where the local temperature can be proportional to the surface gravity, which often has a $1/r$ dependence in simplified $(2+1)$D setups [1, 3]. This choice is thus inspired by known entropic-screen arguments [1] and is not arbitrary.

Remark on the Key Assumptions. By highlighting Eqs. (1) and (2) up front, we emphasize that these are the central postulates of the model: (i) $S(r) \sim r$ follows from the usual "area-law" (now perimeter-law) in $(2+1)$ dimensions, while (ii) $T(r) \sim 1/r$ is chosen in analogy to well-known entropic and holographic screen arguments. Because these assumptions are physically motivated, they serve as a plausible starting point for deriving an entropic force in lower dimensions.

3 Derivation of a $1/r$ Force

We now show that if the total energy $E(r)$ of the boundary satisfies the thermodynamic first-law relation

$$\mathrm{d}E \;=\; T(r)\,\mathrm{d}S(r), \tag{3}$$

then one obtains a potential whose radial derivative is a $1/r$ force.

Combining Eqs. (1) and (2), we have

$$\mathrm{d}E \;=\; \left(\frac{\beta}{r}\right)\,\bigl(\alpha\,\mathrm{d}r\bigr) \;=\; \underbrace{\alpha\,\beta}_{C}\,\frac{\mathrm{d}r}{r}. \tag{4}$$

Here, $C \equiv \alpha\,\beta$ is a constant (which may be taken dimensionless if α has suitable units).

3.1 Energy Function and Radial Force

Integrate Eq. (4) from r_0 to r:

$$E(r) \;-\; E(r_0) \;=\; \int_{r_0}^{r} C\,\frac{\mathrm{d}r'}{r'} \;=\; C\,\ln\!\bigl(r/r_0\bigr). \tag{5}$$

Thus,

$$E(r) \;=\; E(r_0) \;+\; C\,\ln\!\Bigl(\frac{r}{r_0}\Bigr). \tag{6}$$

We may set $E(r_0) = 0$ to define the zero of energy if desired; $r_0 > 0$ is simply a reference scale to avoid singularities in the logarithm as $r \to 0$.

Define the radial "gravitational" force by

$$F(r) \;=\; -\,\frac{\mathrm{d}E}{\mathrm{d}r}. \tag{7}$$

Hence,

$$F(r) \;=\; -\,C\,\frac{1}{r}, \tag{8}$$

a $1/r$ force law. This provides the $(2+1)$D analog of entropic derivations in four dimensions, where an area-scaling entropy yields $F \sim 1/r^2$.

4 Discussion

We have exhibited a minimal route to an entropic force in $(2+1)$D, requiring only two quantum-informational inputs: (i) the perimeter scaling of a boundary's entanglement entropy and (ii) a radial temperature $\sim 1/r$. The resulting energy function $E(r) \propto \ln(r)$ leads directly to an inverse-r radial force, analogous to the usual Newtonian $1/r^2$ law in $(3+1)$D but adapted to one fewer spatial dimension. Once the effective thermodynamic identification $\mathrm{d}E = T\,\mathrm{d}S$ is accepted, little else is required to see this "entropic gravity" mechanism [1, 2, 3, 4].

4.1 Physical Validity and Limitations

In a true BTZ black hole (with negative cosmological constant), the temperature scaling is different from $1/r$. Our use of $T \sim 1/r$ is therefore a flat-space, holographic-screen analogy rather than a literal BTZ black hole property. Moreover, $(2+1)$D Einstein gravity in vacuum does not yield local gravitational forces from point masses; here, the $1/r$ law emerges from boundary entanglement degrees of freedom rather than classical curvature. In that sense, this argument does not conflict with standard theorems regarding $(2+1)$D gravity [8].

It may also be instructive to connect the constant $C = \alpha\,\beta$ with traditional gravitational parameters. In a more realistic $(2+1)$D gravitational scenario, one might identify C with (GM) (in appropriate units) to recover a standard force normalization. However, since our primary interest is the *scaling* of the force, we leave C as a notional constant.

4.2 Significance and Possible Extensions

Despite (or perhaps because of) its simplicity, this toy model strengthens the view that gravity can be treated as an emergent phenomenon arising from quantum entanglement and thermodynamics. It is helpful to see how dimensionality directly affects the "Newtonian" scaling of the entropic force: in $(3+1)$D, one obtains $F \sim 1/r^2$, while in $(2+1)$D the perimeter-law entropy leads to $F \sim 1/r$. This dimensional adaptation underscores the idea that gravitational force laws can follow from how boundary (entangling) surfaces scale with r, possibly hinting at a deeper connection between information geometry and the emergent nature of gravity.

Clarifying how and where the boundary degrees of freedom reside can also be viewed in light of Verlinde's idea that a boundary screen carries the information about the mass enclosed [1], or in terms of an entangling surface in a quantum field vacuum. In practice, one can think of this radial boundary as a "holographic screen" or an "entangling ring" across which states inside and outside are entangled. This interpretation helps justify applying thermodynamic notions (T, S, E) directly to the boundary without invoking a full gravitational framework, consistent with other proposals that treat gravity as an emergent phenomenon of entanglement [3].

It would be interesting to explore whether similar entropic force reasoning could illuminate $(3+1)$D scenarios addressing, e.g., dark energy or modifications of gravity, and also whether lower-dimensional condensed matter analogs might exhibit an "entropic force" in boundary systems. Although speculative, such questions highlight the broader relevance of these minimal derivations to ongoing research in quantum gravity. Even a stripped-down derivation, as shown here, can inform how strongly dimensional considerations influence emergent gravitational behaviors.

Acknowledgments. We thank the broader quantum gravity community for inspiring investigations into emergent forces from entanglement, and for discussions clarifying $(2+1)$D black hole horizon thermodynamics. Any remaining oversights in this brief demonstration are ours alone.

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